ARTICLE VI.

Account of the Observatory and Instruments of the United States Military Academy at West Point; with Observations on the Comet of 1843. *By Prof. William H. C. Bartlett. Read May* 30, 1843.

I HAVE been honoured by the receipt of an invitation to attend the celebration of the one hundredth anniversary of the organization of the American Philosophical Society, at Philadelphia, accompanied by a request from the Committee, to prepare a paper giving an account of the "Observatory and Instruments of the Military Academy," and "Observations on the Comet of 1843."

It will be out of my power to attend, personally, the meeting; and I prepare the paper referred to, more to indicate a desire to contribute to the interest of the occasion, than with any hope that what I have to offer will have this effect.

I ought, in the beginning, to remove a misapprehension under which the Committee seem to be, in regard to the objects of the new building at this place, which is, not unfrequently, called an "Observatory." We have aimed at nothing more in its erection than to provide suitable accommodation for the library of the academy, philosophical apparatus, and such astronomical instruments as were deemed indispensable to illustrate practically our course of astronomy. It supplies the place of one which was, a few years since, destroyed by fire.

It is built principally of granite, taken from the side of the neighbouring mountain, and stands at the south-east angle of the plane of our table land, about one hundred and sixty feet above the level of the river, having an uninterrupted view to the south of about eight miles, and to the north of about four. Fig. *M* represents a perspective view taken from a point about one hundred and fifty yards to the south-west, and Fig. *N* a similar view from a point equally distant to the north-east.* Fig. *P*, Pl. 28, is a horizontal section, midway between the water table and eaves. Fig. *Q*, Pl. 29, is a vertical section parallel to the north front, taken on the broken line AB, CD, EF; and Fig. *R*, Pl. 30, a vertical section perpendicular to this latter front through the centre tower (3.)

The main cell covers a space of one hundred and twenty by sixty feet, and is divided into two equal parts by a partition wall in the plane of the last named section. The

* It has not been thought necessary that plates of these perspective views should accompany the printed paper.

western division (1), with the towers (3), (4), (5), is appropriated to the department of natural and experimental philosophy for purposes of instruction; and the eastern (2) which is in one entire room from the water table to the top of the cell, to the library. The first division has two floors, on the lower of which are the recitation rooms, and on the upper a lecture room in which is the entire apparatus, with the exception of the astronomical instruments. These are in towers prepared expressly for them.

Within the towers, as will be seen by the sections, are masses of masonry, which rise from a bed of coarse gravel at a level twelve feet below the natural surface of the ground, to a height necessary to free the view from all obstruction by neighbouring obstacles. These walls are insulated from those around them, as well as from the floors, stair-ways and ceiling; and are pierced by large openings at different levels to secure a free circulation of air. Immediately to the south of the central tower, is a fourth insulated column (6), Fig. *P*, which terminates on a level near the ridge of the roof, above which the masonry of the enclosing wall is carried about five feet, where it terminates in a kind of parapet; thus forming an open space (7), Fig. *R*, for gazing observations, while the students are occupied in the use of the portable instruments, such as the smaller telescopes, repeating circle, altitude and azimuth instrument, sextant, &c. Access to this place is from the central tower, within which is a small room (9), for the safe keeping of the instruments just named. The mountain to the north may be seen through the door of communication, and an aperture in the column (10), and external wall (11), of the middle tower; thus securing to this position a distant meridian mark to the north as well as to the south. The central tower is surmounted by a travelling dome twenty-seven feet in horizontal diameter, and about seventeen feet high from the spring; and the surface is of a shape which would be generated by the rotation of a circular arc, about a vertical chord somewhat less than the diameter. It is pierced by five window openings near the curb, and an observing slit, two feet wide, extending from a point forty-eight inches above the floor to nearly two feet on the opposite side of the zenith. This last opening is furnished with a range of shutters of nearly equal length which are worked by means of levers and cords independently of each other. The whole dome rests on six twenty-four pound cannon balls, which turn between two cast iron annular grooves, one of which is inverted and attached to the curb piece, and the other to the cell walls of the tower. Motion is communicated to the dome by means of a rack attached to its base, and a stationary pinion, which is turned by a hand-wheel, seven feet in diameter.

Meridian observing slits are made in the flank towers, about twenty inches in the clear. These begin about two feet and a half from the floor and extend through the roof, thus affording an uninterrupted view of the celestial meridian from the southern to the northern point of the horizon. The design and architectural composition of the building, which is of the Elizabethan style of Gothic, and the effect of which is considered very good, is due to Major Delafield, the Superintendent of the institution.

THE EQUATORIAL.

The only fixed instrument in place, is in the central tower. It is an equatorial by Mr. Thomas Grubb, of Dublin, which is believed to possess some merit. Its object is to measure by direct observation the apparent declination and hour angle of any heavenly body. By means of the latter and the sidereal time of observation, the apparent right ascension, which is equal to their sum, becomes known. The instrument consists, essentially, of two axes of motion at right angles to each other, two graduated circles, one attached to each axis at right angles to its length, and a supporting frame. When the instrument is adjusted for use, one axis is parallel to that of the earth, and the other to the plane of the equator, the former being called the polar and the latter the declination axis.

In principle, Mr. Grubb's instrument resembles that of Fraunhofer. In the arrangement of its parts, it is, however, different, and very simple. Fig. *S* represents a perspective view of this instrument taken from a point to its north-east, the telescope being in the meridian. Fig. *T* exhibits a meridian section, the declination axis being in this plane, and the telescope directed to the pole. The polar axis, (1) Fig. *T*, is about forty inches long between the bearings, is of a conical shape, terminating towards the vertex in a well turned steel pivot about three-eighths of an inch in diameter, and at its base in an accurately turned cylinder, one inch in length, and about eight inches in diameter. It is hollow for a distance of more than three-fourths of its length from the larger end, and is mounted in the centre of two acute hollow conic frustrums (2), (3), which are united to each other by screw-bolts, as represented at (4), so as to have a common axis. The diameter of the largest base is twenty-two inches, and that of the smallest ten and a half. Connected with a flanch (9), at the largest base, are two substantial pieces of wrought iron (5), united in a crucial form at right angles to each other, and perforated at the place of union, which is in the common axis of the frustrums. This perforation receives a cylindrical box of iron with a large bearing head (6), and this latter a solid cylindrical piece of Brazilian metal, terminating at one end in a bearing head (7), in which is a small cavity for the reception of the lower end of the polar axis, and at the other in a screw for a nut, which keeps it in place. This latter cylindrical piece is smaller than the opening through the box (6), and may be moved laterally by means of four small screws (8), which enter the box at right angles to each other. By this device, the more delicate adjustments of the polar axis are made. The collar, or upper bearing surface of the polar axis, rests upon two pieces of Brazilian metal (10), Fig. *C*. which are let into the inner surface of the smaller conic frustrum, at its smaller base. These pieces are about three quarters by one inch on the rubbing surface, and of a proper shape to receive the cylindrical form of the axis. They are 90° apart, each being 45° from the meridian; and are relieved from a great portion of the pressure, by a friction roller (11). The polar axis has, at the larger end, a square flanch, projecting outward so as to measure eight and a half inches on a side; the edge of this flanch is seen at (15), Fig. *S*, and to its lower surface, the hour circle (13) is attached. The hour circle is of brass, and in shape resembles, when in place, an inverted shallow cylindrical cup, with a circular hole in its base just large enough to receive the polar axis. The graduation is on a band of

silver sunk into its outer surface, which is twelve inches in diameter, and reads, by the aid of three verniers, to a second of time.

Upon the square flanch, at the top of the polar axis, rests a cast iron frame for supporting the declination axis. It consists of a cubical base eight and a half inches on the edge, perforated on opposite faces with which are united at their larger bases two short hollow conic frustrums (16). Attached to one end of this frame is a circular flanch (17), fifteen inches in diameter, with a channel in its circumference to receive a friction hoop for the tangent screw (22), Fig. *S*, of the telescope and declination circle. Two pieces of Brazilian metal, similar to those shown in Fig. *C*, are let into the concave surface of the opening in this circular flanch, and serve as Y bearings for one end of the declination axis. The other end of the frame terminates in a short flanch against which is supported, by strong screws, a circular plate (19), of Brazilian metal. This plate has a circular apperture large enough to receive freely the declination axis; and to secure the proper bearing points, the lower edge of the aperture is cut away, as shown at (20), Fig. *D*, thus converting the circular into a Y bearing for the other end of the axis. The screw holes in this plate are of an oblong shape, to admit of a motion in the plate itself in the direction of the polar axis. This motion is communicated by means of a device, shown at (21). By this, the declination axis is adjusted.

This axis (23) is represented out of its bearings, because the section is made where they are not found. One end of this axis has a strong square flanch eight inches on a side, and to this the cradle of the telescope is attached by strong screw bolts, of which the head of one is seen at (26), Fig. *S*. The diameter of this axis at the bearing surface nearest the telescope is four inches, and at the other, a little more than three. About three-fourths of an inch from the bearing plate (19), the axis is contracted to form a shoulder to receive the declination circle (24), which is secured by two strong steel screws; and beyond this, the axis projects about eleven inches to receive the counterpoise weights (27). To prevent all motion in this axis in the direction of its length, a steel roller (34), about one inch in diameter, and twenty-eight inches long, is held in the centre of the polar axis by two perforated steel bars (35), (36), which are firmly attached to the polar axis, and within which the roller turns with the utmost freedom, but without play. At the upper end of the roller is a circular head, whose diameter is just equal to the width of an annular groove in the declination axis within which it works. The axis is pressed into its bearings by a friction wheel which enters the cubical box at the top, and is secured in its place by a stiff frame and screw (29); and the whole frame is securely attached to the head of the polar axis by strong screw bolts (30), represented in dotted lines.

The declination circle (24), which is fifteen inches in diameter, is also of brass, and the graduation is on a band of silver sunk into its outer rim. It has two verniers (32), 180° apart, by which the smallest count is fifteen seconds of space, and two reading micrometers (31), by which the smallest count is reduced to one second.

The clock-work is represented at (37), Fig. *S*. It is put in communication with the instrument by means of a circular sector (38), whose centre of curvature is in the polar axis, and whose circumference is provided with teeth that work into an endless screw, of which the axis is seen at (48). To the arm (40) of this sector, one of the nuts (41) of the

tangent screw of the hour circle is attached. The arrangement of the tangent screw is shown at (39) in Fig. *T*. In this figure (44) is a circular plate, seventeen inches in diameter, with an aperture in the centre large enough to receive the polar axis, to which it is firmly attached by a key bolt (43). In the circumference of this circle is a groove which receives the friction band with its clamp screw (42), and to this band is attached the arm (46) which carries the nut of the tangent screw (39). Against the upper surface of this circle, the sectoreal arm (40), represented in Fig. *E*, is held by a small truncated conical plate (45) Fig. *T*, which fits into a chamfered aperture (47) in the arm, and is securely attached by screws to the clamp circle. When the circle is clamped, and the teeth of the sector in gear, the hour circle can move only with the clock, or by the turns of the tangent screw; but when unclamped, the hour circle is moved freely by the hand applied to the telescope, while the sector is in connexion with the clock; and the instant the hand is removed, the hour circle takes up the clock movement, in consequence of the friction between the surface of the clamp circle and sector. This device is very convenient, as it saves the labour and time of unclamping and clamping in transferring the telescope from one object to another. The axis (48), Fig. *S*, of the endless screw, passes through a circular aperture in the centre of the wheel (49), and the arrangement is such, that by means of a small pin, the wheel and axis may be firmly united or rendered independent of each other, so that when the sector is exhausted by the motion of the clock, it is renewed by throwing the wheel out of gear with the axis, and reversing with the hand, by means of a winch-key, the motion of the endless screw.

The rate of the clock is regulated by a centrifugal governor, the balls of which are attached to the ends of a horizontal piece, at the top of the axis of motion, and when at rest are supported by a second cross piece (51). Connected with the supporting rods of the balls, are two brass arms that carry each a small box-wood screw (52), whose axis is perpendicular to a horizontal friction plate (53), with which they are brought in contact by the recession of the balls from the axis during the motion. By unscrewing these, the rotation is accelerated,—by screwing them up, retarded. The whole instrument is placed upon a solid block of granite, of the shape shown in Figs. *S* and *T*, to which it is attached by heavy screw bolts sunk into the inclined surface (60), which is parallel to the equator, and secured by molten sulphur. The surface (61) is horizontal.

The telescope, by Lerebours of Paris, is a refractor whose solar focal distance is eight feet and aperture six inches. It has a fine position micrometer (56), furnished with an illuminating apparatus for bright lines and dark field, the lines being illuminated on both sides. The diameter of the position circle is   inches; the graduation is on a silver band, and reads by the aid of two verniers as low as thirty seconds. The micrometer is also by Grubb. (57) is a small finder, and (58) a mass of lead attached to the cradle to counterpoise the telescope.

ADJUSTMENTS.

The stone block upon which the instrument rests having been carefully cut to a frame prepared for the stone-cutter, was put in place by the aid of a small theodolite, whose axis of collimation was carefully adjusted by Polaris to the meridian. A well defined

black line was drawn upon the equatorial and horizontal surface of the stone, perpendicular to the line of intersection of these surfaces at its middle point, and the latter surface being kept horizontal by means of a large spirit level, the stone was moved till the traces were brought into the meridian, as indicated by a sweep of the telescope of the theodolite. The instrument was then put up.

The following conditions must be fulfilled by an equatorial in perfect adjustment.

1. The polar axis must be parallel to that of the heavens.
2. The declination axis must be parallel to the equator, or perpendicular to the polar axis.
3. The line of collimation of the telescope must be perpendicular to the declination axis.
4. The index of the declination circle must point to zero, when the line of collimation is parallel to the equator.
5. The index of the hour circle must point to zero, when the line of collimation is in the meridian of the place.

M. Littrow has given in the Mem. of the Ast. Soc. Vol. II. part I. p. 45, formulæ for the determination of the true apparent place of a heavenly body from the place as indicated by an equatorial out of adjustment. These were improved by M. Kriel, Mem. of the Ast. Soc. Vol. IV., p. 495. The adjustments have been made by these formulæ, which are

$$s=\sigma+\Delta\,\sigma+\lambda.\ \sin\,(\phi-s).\ \tan\,\delta+\mu\,\tan\,\delta+\nu\ \sec.\ \delta\,.\,.\ (1)$$
$$p=\pi+\Delta\,\pi+\lambda.\ \cos\,(\phi-s).\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\ (2)$$

In which s is the true, and σ the instrumental hour angle, corrected for refraction; p the true, and π the instrumental polar distance corrected for refraction; $\Delta\,\sigma$ and $\Delta\,\pi$ the index errors of the hour and declination circle respectively; λ, the distance, in arc, between the pole of the heavens and that of the instrument; ϕ the hour angle of the instrumental pole, estimated from the meridian to the west; μ is the difference between 90° and the angle which the declination axis makes with the polar; ν the difference between 90° and the angle which the line of collimation makes with the declination axis; and δ the declination of the body. The adjustments of the several parts of the instrument, with respect to each other, were first made by means of terrestrial objects. These are always at hand, and vary in declination, from zero to the complement of the latitude of the place, which, in the present case, is 48° 36′; the former being in the instrumental east and west, and the latter in the instrumental north and south.

INDEX ERROR OF DECLINATION CIRCLE.

The telescope was directed to a well defined object in any part of the horizon in the direct and reverse position of the instrument, and equation (2) gave

$$p=\pi+\Delta\,\pi+\lambda.\ \cos\,(\phi-s)$$
$$p=\pi'-\Delta\,\pi+\lambda.\ \cos\,(\phi-s)$$

whence we obtain, by taking the second from the first,

$$\Delta\pi = \frac{\pi - \pi'}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

by which the verniers were corrected.

THE LINE OF COLLIMATION.

The declination circle was placed at zero, after correcting the verniers, (the instrument reads declinations,) and the telescope directed to some distant point to the east or west, in the direct and reversed position of the instrument. In equation (1), we had

$$\delta = 0;$$

and because the graduation of the hour circle is numbered from 0° to 360°, or from 0 to 24^h.

$$s = \sigma + \Delta\sigma + \nu,$$
$$s = \sigma' + \Delta\sigma - \nu;$$

subtracting the second from the first we get

$$\nu = \frac{\sigma - \sigma'}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This error was applied with its proper sign to the last reading, to which the instrument was then set, and the line of collimation brought to bear on the object by the screws which attach the cradle to the declination axis.

THE DECLINATION AXIS PERPENDICULAR TO THE POLE.

The telescope was directed to a point directly north or south, to get the greatest declination, in the direct and reverse position; and because by the last adjustment, $\nu = 0$, equation (1) gave

$$s = \sigma + \Delta\sigma + \lambda . \sin(\phi - s) . \tan\delta + \mu . \tan\delta.$$
$$s = \sigma' + \Delta\sigma + \lambda . \sin(\phi - s) . \tan\delta - \mu . \tan\delta.$$

Taking the difference and reducing, we had

$$\mu = \frac{\sigma - \sigma'}{2\tan\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Setting the instrument to the last reading corrected by $\frac{\sigma - \sigma'}{2}$, the line of collimation was brought back to the object by the adjusting screw (21), connected with one end of the declination axis.

THE POLAR AXIS OF THE INSTRUMENT PARALLEL TO THAT OF THE HEAVENS.

A well known star was observed in quick succession on the meridian, in the direct and reversed position of the instrument. The declination reading, corrected for refraction, gave in equation (2)

$$p=\pi+\Delta\,\pi+\lambda.\ \cos\phi,$$
$$p=\pi'-\Delta\,\pi+\lambda.\ \cos\phi;$$

whence

$$\lambda\cos\phi=p-\frac{\pi+\pi'}{2}\quad\ldots\ldots\ldots\quad(6)$$

The first member, being the projection of the arc λ on the meridian, is the arc by which the pole of the instrument is too high or too low. The foot of the polar axis being moved through this distance by estimation, the declination circle was then set to the declination, corrected for refraction, of a second star soon to come to the meridian, and the telescope directed to it, the clock put in motion, and as the star culminated, as indicated by a chronometer, the adjusting screws (8), which are in the meridian at the foot of the polar axis, were turned till the line of collimation was brought to the star.

Another star was observed in quick succession in the direct and reverse position of the instrument, when six hours to the east or west of the meridian, in which case

$$s=90°,$$

and formula (2) gave

$$p=\pi+\Delta\,\pi+\lambda.\ \sin\phi,$$
$$p=\pi'-\Delta\,\pi+\lambda.\ \sin\phi;$$

whence

$$\lambda.\ \sin\phi=p-\frac{\pi+\pi'}{2};\quad\ldots\ldots\ldots\ldots\quad(7)$$

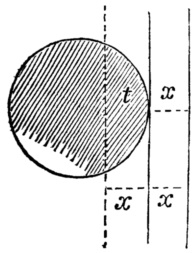
the first member, being the projection of the arc λ on a declination circle at right angles to the meridian, is the displacement of the pole of the instrument from this latter plane, and was hence known. This error being treated in manner similar to the preceding by the screws (8) which are perpendicular to the meridian, the adjustment of the polar axis was completed.

The instrument when first put together being very much out of adjustment, terrestrial objects, as already remarked, were taken as being more convenient for the first approximations. Celestial objects in similar positions were afterwards used for all subsequent ones.

VALUE OF THE MICROMETER REVOLUTION.

This was obtained by the usual process of taking the transits of stars near the equator and the distances of well known double stars. But the process which gave results agreeing with each other as well as any, is one which occurred to me while observing

the solar spots; and as I have not met with it before, I have thought it would not be out of place to give it here.



Let s denote the apparent diameter of the sun in seconds of space, x the apparent distance between the parallel wires of the micrometer expressed in the same unit, t the time of the sun's diameter passing each wire, supposed perpendicular to the equator, and t' the time required for the same limb to pass from one wire to the other. Then assuming that the motion of the sun in right ascension is uniform during the transit over each wire, will

$$s : x :: t : t';$$

$$x = s \cdot \frac{t'}{t};$$

whence calling ν the value of one division on the screw head, and n the number of these divisions which mark the separation of the wires, we have

$$\nu = \frac{x}{n} = \frac{s}{n} \cdot \frac{t'}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

EXAMPLE.

March 24*th*, 1843.

1st Limb.	$11^h\ 25^m\ 40^s.5$	(1)	A		1st Limb.	$11^h\ 32^m\ 12^s.$	(1)	B
	$11\ \ 26\ \ 34.$	(2)				$11\ \ 33\ \ 04.5$	(2)	
2d Limb.	$11\ \ 27\ \ 54.$	(1)′			2d Limb.	$11\ \ 34\ \ 24.5$	(1)′	
	$11\ \ 28\ \ 42.5$	(2)′				$11\ \ 35\ \ 13.$	(2)′	

$$A \begin{cases} (1)' - (1) = 2^m\ 13^s.5 \\ (2)' - (2) = 2\ \ 08.5 \end{cases}$$

$$B \begin{cases} (1)' - (1) = 2^m\ 12^s.5 \\ (2)' - (2) = 2\ \ 08.5 \end{cases}$$

$$4)\,8\ \ 43 = 2^m\ 10^s.75 = 130^s.75 = t.$$

$$A \begin{cases} (2) - (1) = 53^s.5 \\ (2)' - (1)' = 48.5 \end{cases}$$

$$B \begin{cases} (2) - (1) = 52^s.5 \\ (2)' - (1)' = 48.5 \end{cases}$$

$$4)\,203 = 50^s.75 = t'.$$

From Naut. Alm. $s = 1925''.68$.

The wires were separated by a distance of ten revolutions in each screw, and the heads having one hundred divisions each,

$$n = 2000.$$

These values substituted in eq. (8) give

$$\nu = \frac{1925''.68}{2000} \times \frac{50^s.75}{130^s.75} = 0''.\,37367.$$

THE COMET OF 1843.

The instrument just described was hardly in working condition, before the announcement of a comet within our system gave hopes of a speedy opportunity to test its qualities.

The weather was unfavourable till the 24th of March, when the first observations upon the new visiter were made; the last were on the 2d of April. Of the intermediate days, the only ones admitting of observation were the 25th and 29th of March. I have taken those of 25th and 29th of March, and of 2d April, to compute the elements of the orbit.

The observations were commenced by the usual simple and accurate method of measuring differences of right ascension and declination of the comet, and certain small stars, which were in the field at the same time; but recollecting that I had no catalogue more extensive than that of the Royal Astron. Society, in which the stars in question could not be found, I was obliged to have recourse to another method.

It is obvious that *the difference of the instrumental places of two bodies, corrected for the change of instrumental errors arising from a change of position, is equal to the difference of their true places.* This corrected difference may be found from Kriel's formulæ (1) and (2), as soon as λ, ϕ, μ and ν are known. If then, some well known star be observed on the same evening with the comet, the true place of the latter becomes known, since that of the former is obtained from the catalogue, and the quantity by which it differs from the place of the comet, by observation. To keep the instrument as nearly as possible in the same bearings during the observations on both bodies, *α ceti*, in the neighbourhood of the comet, was selected and used throughout.

TO FIND ϕ AND λ.

The co-ordinates of a known star, corrected for refraction, when on the meridian, and six hours to the east or west, being substituted in equations (6) and (7), we obtain by squaring, adding, and taking square root,

$$\lambda = \pm \sqrt{\left(p_{\prime} - \frac{\pi_{\prime} + \pi_{\prime\prime}}{2}\right)^2 + \left(p' - \frac{\pi' + \pi''}{2}\right)^2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and dividing eq. (7) by eq. (6)

$$\text{Tan } \phi = \frac{p' - \frac{\pi' + \pi''}{2}}{p_{\prime} - \frac{\pi_{\prime} + \pi_{\prime\prime}}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

TO FIND μ AND ν.

Denote by Æ, the right ascension of a star, and by t' the sidereal time of observation, then will

$$s = t' - Æ.,$$

and eq. (1), for the same star observed in quick succession in direct and reversed position, gives

$$s = (t' - Æ) = \sigma' + d\sigma + \chi \sin(\phi - s) \tan \delta' + \mu \tan \delta' + \nu \sec \delta'.$$
$$s' = (t'' - Æ) = \sigma'' + d\sigma + \lambda \sin(\phi - s) \tan \delta' - \mu \tan \delta' - \nu \sec \delta'.$$

Taking the first from the second and reducing, we have

$$(t'' - t') - (\sigma'' - \sigma') = -2\,\mu \tan \delta' - 2\,\nu \sec \delta'. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and for a second star differing considerably from the first in declination,

$$(t_{,,} - t_{,}) - (\sigma_{,,} - \sigma_{,}) = -2\,\mu \tan \delta'' - 2\,\nu \sec \delta''. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

in which μ and ν are the only unknown quantities.

EXAMPLE.

To illustrate, I take the stars observed on the 3d of April.

Object.	Face	Time by Chronometer.	Chrono. Interval.	Sidereal Interval.	Hour Circle.	Hour Circle Interval.	Dec. Circle.	Refr.	Instrumental Declination.	Naut. Alm. True Declination.
				$(t_{,} - t)$		$(\sigma' - \sigma)$				
γ Ursæ Majoris.	N.	4h41m42s.	8m00s.	8m01s.31	5h56m16s.	8m05s.	54°33′45″.	−40″.89	54°33′04″.1	54°33′52″.48
	S.	4 49 42 .			18 04 21 .		54 33 45 .			
ϵ Hydræ.	W.	7 35 01 .5	6 26 .	6 27 .05	17 57 10 .	6 38 .	7 01 00 .	−39 .89	7 00 35 .1	6 59 16 .3
	E.	7 41 27 .5			6 03 48 .		7 01 30 .			
θ Ursæ Majoris.	E.	8 18 14 .	10 26 .	10 27 .7	5 56 51 .	10 12 .	52 24 30 .	11 .35	52 24 23 .5	52 23 18 .3
	W.	8 28 40 .			18 07 03 .		52 23 55 .			

The data furnished by this table in equations (9) and (10), give

$$\lambda = \sqrt{(48.4)^2 + (72)^2} = 86''.75 = 1'26''.75.$$

$$\text{Tan } \phi = \frac{48.4}{72} = \text{Tan } 33°54'35''.$$

$$\phi = 33°54'35''.$$

and in equations (11), (12), give, regarding face west as the reversed position,

$$0.24\,\mu + 2.02\,\nu = -11^s$$
$$2.60\,\mu + 3.28\,\nu = -15^s$$

whence

$$\nu = -5^s.56. \quad . \quad . \quad . \quad . \quad . \text{ Face E.}$$
$$\mu = 0^s.95. \quad . \quad . \quad . \quad . \quad . \quad . \text{ Face E.}$$

These values being substituted in equations (1) and (2), the Comet was wrought out as in the following example for the 29th of March.

Data from observation.

Date	Object,	Sid. Time	Hour Circle	Dec. Circle
1843,	α ceti	$8^h\ 00^m\ 16^s.8$	$5^h\ 06^m\ 18^s.$	$3°\ 36'\ 15''.$
March 29th.	Comet.	8 31 27 .4	4 36 28.	—6 40 15 .

Columns fourth and fifth were corrected for refraction by the following formulæ, the thermometer and barometer indicating about mean refraction, namely,

$$\text{Refr. in } \text{Æ} = 57'' \frac{\text{Tan } s. \sin \psi}{\text{Cos } \delta. \sin (\psi + \delta)}$$

$$\text{Refr. in Dec.} = 57''. \cot (\psi + \delta).$$

$$\text{Tan } \psi = \cos s. \cot L.$$

In which L is the latitude of the place, being in this instance, 41° 23′ 31″; and s the hour angle of the instrument; the values of σ and π, for both bodies are therefore known: and employing σ for s in equation (1) we have for

α ceti	$\sigma = 5^h\ 06^m\ 30^s.9$	comet	$\sigma = 4^h\ 36^m\ 41^s.01$
$\lambda \sin (\phi - s) \tan \delta$	$= -0\ 00\ 00.24$	$\lambda \sin (\phi - s)$, tan .	$\delta = 0\ 00\ 00.39$
$\mu \tan \delta$	$= -0\ 00\ 00.16$	$\mu \tan \delta$	$= 0\ 00\ 00.11$
$\nu \sec. \delta$ . . .	$= 0\ 00\ 07.56$	$\gamma \sec. \delta$	$= 0\ 00\ 06.81$
	$s = 5^h\ 06^m\ 38^s.16$		$s' = 4^h\ 36^m\ 48^s.31$

whence

$$s' - s = 23^h\ 30^m\ 10^s.15 = t' - t + \text{Æ} - \text{Æ}'.$$
$$0\ 31\ 10.60 = t' - t.$$

$$22^h\ 58^m\ 59^s.55 = \text{Æ} - \text{Æ}'.$$

Naut. Almanac, . . . 2 54 05 .40 = true Æ of a ceti.

$$3^h\ 55^m\ 05^s.85 = \text{true Æ}' \text{ of comet.}$$

And from equation (2) we have for

α ceti, π . . .	= 86° 26′ 36″.7	comet π'	= 96° 43′ 42″.70
$\lambda. \cos (\phi - s)$.	= 0 01 03 .72	$\lambda. \cos (\phi - s)$ . .	= 0 01 10 .82
	86° 27′ 40″.42		96° 44′ 53″.52

whence,

$\pi' - \pi$ = 10° 17′ 13″.10,
P = 86 31 45 . true polar dist. of α ceti.

96° 48′ 58″.10 true polar dist. of comet.
δ = — 6 48 58 .10 true declination of comet.

In this way the following places were obtained for the comet.

March 25,	Æ	$3^h\ 35^m\ 20^s.14$;	Dec. $-7°\ 48'\ 46''.6 = \delta$.
" 29,	"	3 55 05 .8;	" $-6\ 48\ 58 = \delta$.
April 2,	"	4 11 51 .95;	" $-5\ 58\ 43\ .1 = \delta$.

which being converted into longitudes and latitudes, and cleared from the effects of aberration by the usual formulæ for a fixed star, gave

March 25,	longitude . . . . . 49° 17′ 52″;	latitude . . — 26° 18′ 11″.
" 29,	" . . . . . 54 52 35 ;	" . . — 26 32 35″.
April 2,	" . . . . . 59 37 38 ;	" . . — 26 37 42 .

The portion of the aberration due to the proper motion of the comet was applied to the time, according to the method of M. Gauss. The correction in the place of the earth for the effect of parallax, was disregarded in consequence of its small value, the first curtate distance being greater than unity.

The method by which the following elements were computed was that of Dr. Olbers, as given by Dr. Bowditch in the appendix to his commentary on the "*Mecanique Celeste.*"

Longitude of the ascending node . . .	357° 41′ 49″.
Inclination, ...	36 41 48 .
Longitude perihelion, ...	261 31 47 .
Perihelion distance, ...	0.053774.
Perihelion passage, Greenwich mean time, February, .	26^d. 6018.

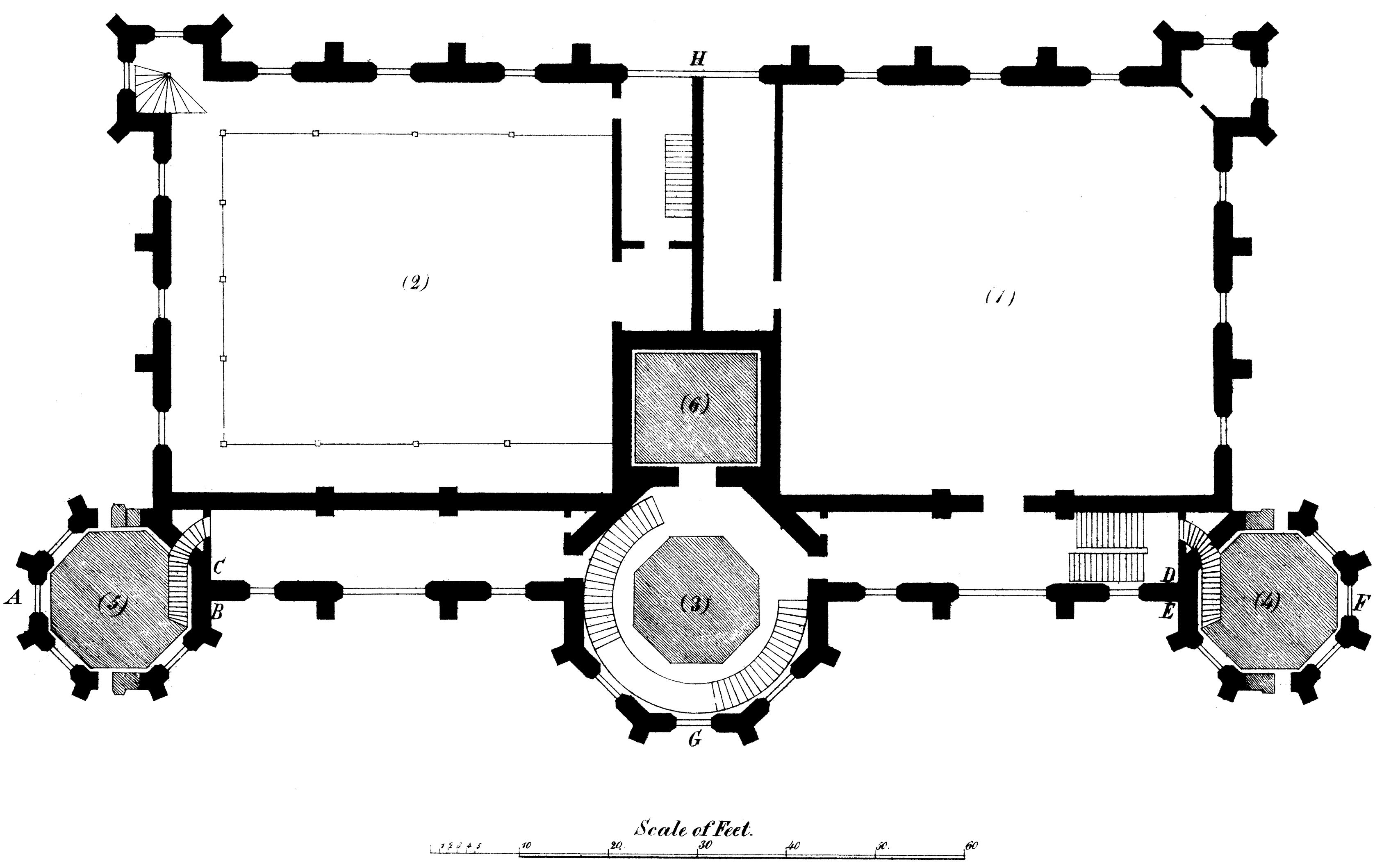
Motion Retrograde.—Distance from the earth on the evening of 29th March, ... 107,002,000 miles.

Approximate diameter of the nebulous envelope, ... 36,800.

I am indebted to Prof. Church for having kindly undertaken an independent computation of the elements as a check upon my own, and for having assisted with Lieutenant Roberts in making the observations. The perspective views are by Lieutenant Richard Smith, and the sections of the building by Lieutenant Eaton.

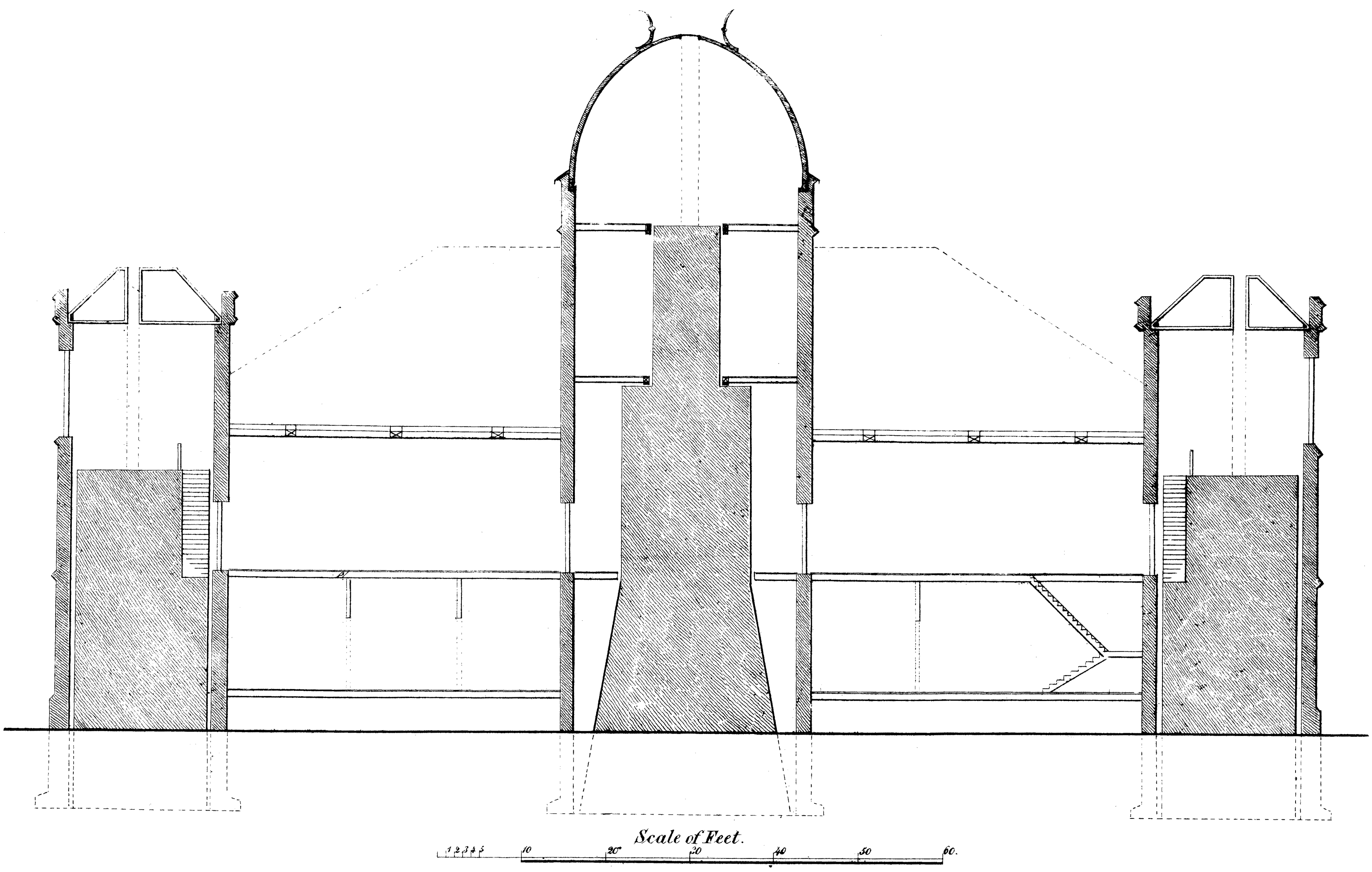
WM. H. C. BARTLETT.

Fig. P.



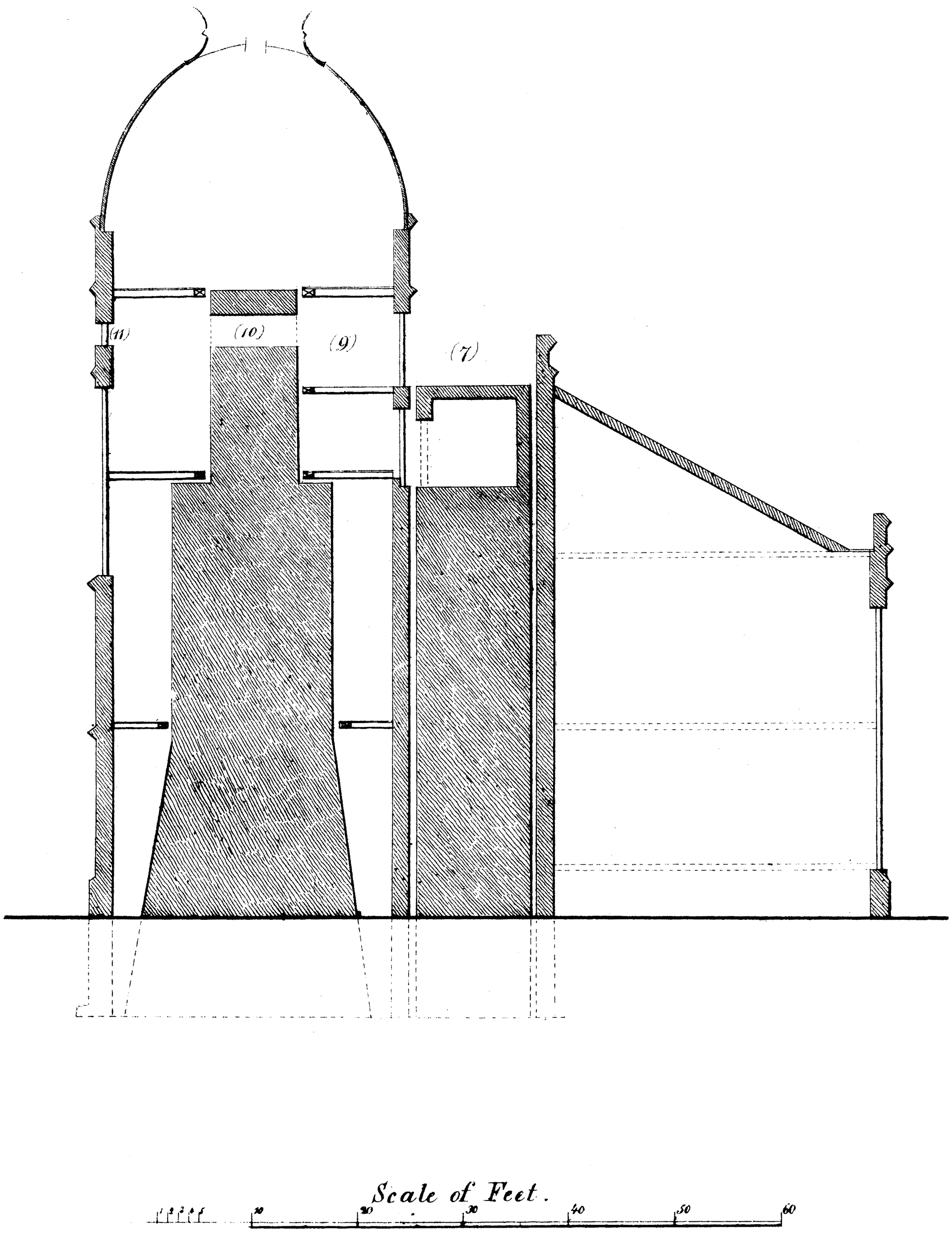
Ground plan of the building for the Library & Philosophical Apparatus West Point.

Fig. Q.



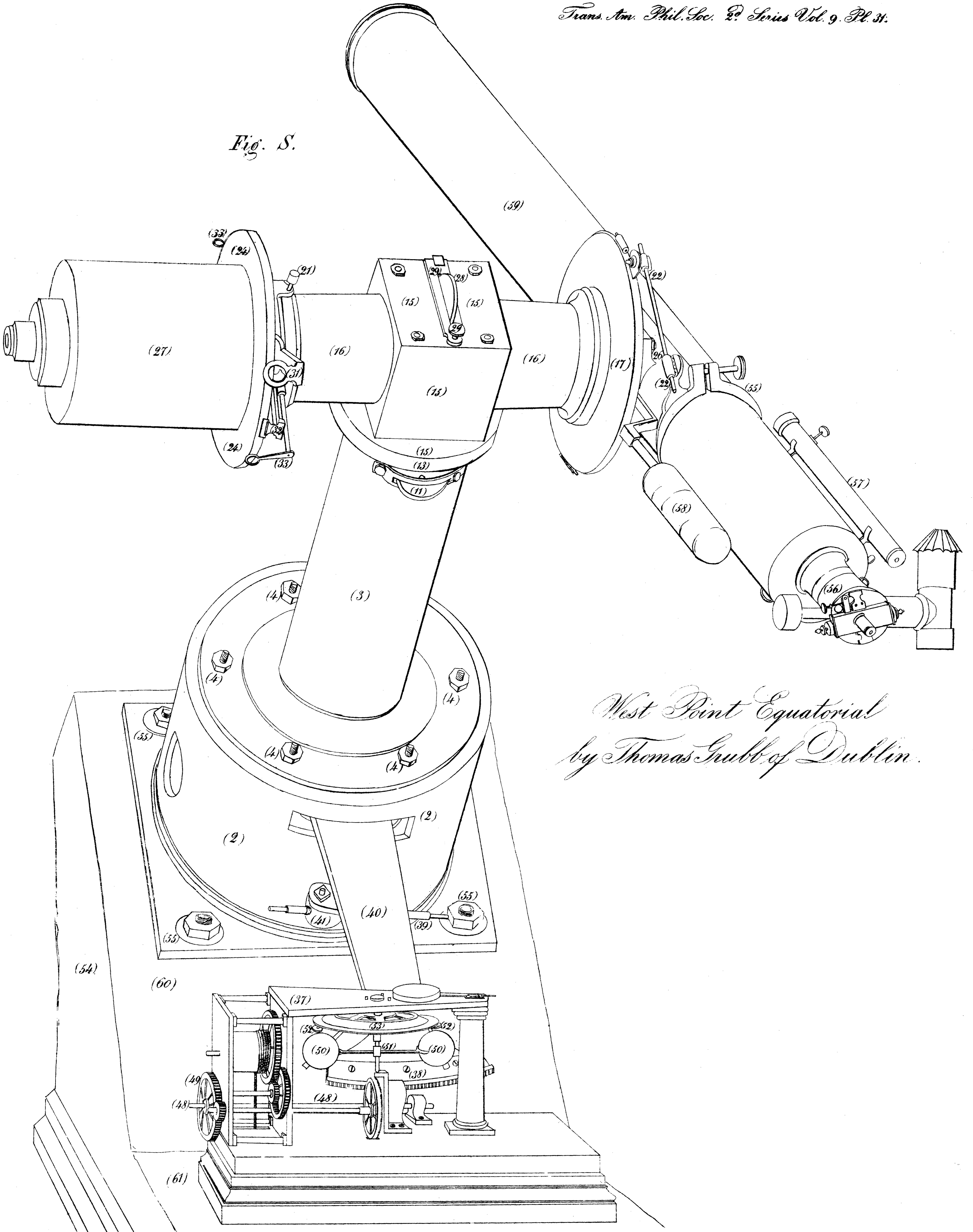
Section through the towers on the North front

Fig. R.

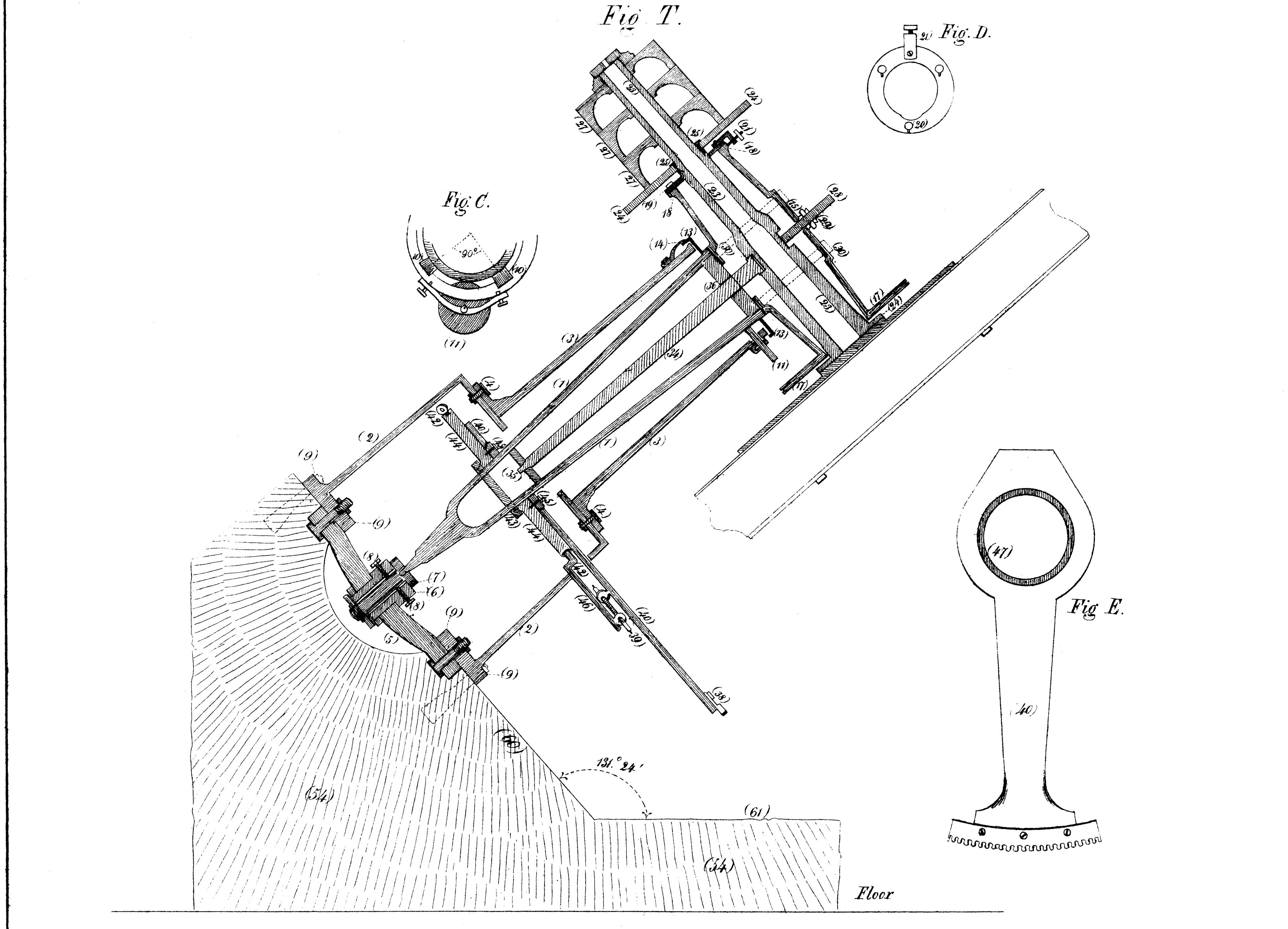


Section through the middle tower, perpendicular to the North front.

Fig. 8.



West Point Equatorial
by Thomas Grubb of Dublin.



Meridian Section of the Equatorial.